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UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 8

Question Paper Code : UM9274

KEY

1	2	3	4	5	6	7	8	9	10
D	C	A	C	A	C	A	C	B	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	C	D	B	B	D	C	D	C	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	A	C	D	D	C	A	C	A	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,B	B,D	A,B,C,D	B,D	A,B,C,D	B	D	D	B	A
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
B	C	A	C	A	B	A	C	C	C

EXPLANATIONS

MATHEMATICS - 1 (MCQ)

1. (D) Let $x = n^2$, $n \geq 0$

then $(n + 1)^2 = n^2 + 2n + 1$

$= x + 2\sqrt{x} + 1$

2. (C) $\angle B = 90° \Rightarrow \angle D = 90°$ and $\angle A = \angle C = 90°$

$\therefore$ ABCD is a rectangle.

3. (A) Side of the first square = $(2x - 1)$ m

$\Rightarrow$ Area = $(2x - 1)^2$ m^2

Side of the second square = $(5x + 4)$ m

$\Rightarrow$ Area = $(5x + 4)^2$ m^2

According to the problem,

$9(2x - 1)^2 = (5x + 4)^2$

$\Rightarrow [3(2x - 1)]^2 - (5x + 4)^2 = 0$

$\Rightarrow [3(2x - 1) + 5x + 4]$

$[3(2x - 1) - 5x - 4] = 0$

$\Rightarrow (6x - 3 + 5x + 4)(6x - 3 - 5x - 4) = 0$

$\Rightarrow$ $(11x + 1)(x - 7) = 0$

$\Rightarrow x = \frac{-1}{11}$ or 7

Dimensions of the plots cannot be negative.

Hence $x = 7 \Rightarrow 2x - 1 = 2(7) - 1 = 13$ m

$5x + 4 = 5(7) + 4 = 35 + 4 = 39$ m

4. (C) Four numbers are in proportion if First × Fourth = Second × Third

Let 'x be added to each of the given numbers to make the numbers proportionate.

Then,

$(5 + x)(27 + x) = (9 + x)(17 + x)$

$\Rightarrow 135 + 32x + x^2 = 153 + 26x + x^2$

$\Rightarrow 32x - 26x = 153 - 135$

$\Rightarrow 6x = 18$

$\Rightarrow x = 3$

5. (A) Let the length of the side of the smaller square be 'x' m. Then its area is x^2 sq. m.

$\therefore$ The length of the side of the larger square = $(x + 4)$ m

$\Rightarrow$ Its area = $(x + 4)^2$ sq. m

= $x^2 + 8x + 16$ sq. m

Given that the sum of their areas is 208 sq. m.

$\Rightarrow x^2 + x^2 + 8x + 16 = 208$

$\Rightarrow 2x^2 + 8x + 16 = 208$

$\Rightarrow x^2 + 4x + 8 - 104 = 0$

$\Rightarrow x^2 + 4x - 96 = 0$

$\Rightarrow x^2 + 12x - 8x - 96 = 0$

$\Rightarrow x(x + 12) - 8(x + 12) = 0$

$\Rightarrow x = -12$ or 8

Since the side of a square cannot be negative, the side of the smaller square is 8 m.

$\therefore$ The side of the larger square is 12 m.

6. (C) $P\left(1 + \frac{11}{100}\right)^2 - P - \frac{P \times 11 \times 2}{100} = ₹\ 363$

$P\left(\frac{111}{100}\right)^2 - P - \frac{11P}{50} = ₹\ 363$

$\frac{12321 \times P}{10,000} - P - \frac{11P}{50} = ₹\ 363$

$\frac{12321P - 10,000P - 2200P}{10,000} = ₹\ 363$

$\frac{121P}{10,000} = ₹\ 363$

P = ₹ 30,000

7. (A) $27^{64} = (3^3)^{64} = 3^{192}$

$9^{100} = (3^2)^{100} = 3^{200}$

$81^{49} = (3^4)^{49} = 3^{196}$

$3^{198} = 3^{198}$

$\therefore$ 9^{100} is the greatest

8. (C) There are 21 consonants among 26 alphabet.

$\therefore$ Probability of consonant = $\frac{21}{26}$

9. (B) $\Rightarrow a = \frac{1}{a}$

Number = its reciprocal

$\Rightarrow 1, -1$

$1 + \frac{1}{1} = 2$

$-1 + \frac{1}{-1} = -2$

10. (C) $\sqrt[3]{49} \times \sqrt[3]{-448}$

$= \sqrt[3]{(49) \times (-448)}$

$= \sqrt[3]{49 \times (-7)(64)}$

$= \sqrt[3]{(-343)(64)} = \sqrt[3]{(-7)^3(4^3)}$

= −28

11. (C) $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$=\left(x^2-\frac{1}{x^2}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$=\left[\left(x^2\right)^2-\left(\frac{1}{x^2}\right)^2\right]\left(x^4+\frac{1}{x^4}\right)$

$=\left(x^4\right)^2-\left(\frac{1}{x^4}\right)^2$

$=x^8-\frac{1}{x^8}$

12. (C) Option 'C' is true.

13. (D) Volume each small metalic cube = a^3 = $(2\text{ cm})^3$ = 8 cm^3

Let 'n' cubes dropped in the tank

$\therefore$ Volume of 'n' cubes = Volume of risen water

$n \times 8cm^3 = 8 \times 5 \times 4\ cm^3$

$n=\frac{8\times 20\ cm^3}{8cm^3}=20$

14. (B) $\frac{de\times bc}{ab\times cd}=\frac{5\times 2}{3\times 4_2}$

$\frac{e}{a}=\frac{5}{6}$

15. (B) $\frac{a+2\sqrt{ab}+b}{\sqrt{a}+\sqrt{b}}=\frac{(\sqrt{a})^2+2\sqrt{a}\times\sqrt{b}+(\sqrt{b})^2}{(\sqrt{a}+\sqrt{b})}$

$=\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}+\sqrt{b})}$

$=(\sqrt{a}+\sqrt{b})$

16. (D) $\sqrt{32.5^2+18.5^2-17.5^2-31.5^2}$

$=\sqrt{(32.5)^2-(31.5)^2+(18.5)^2-(17.5)^2}$

$=\sqrt{(32.5+31.5)(32.5-31.5)+(18.5+17.5)(18.5-17.5)}$

$=\sqrt{64(1)+36(1)}\ =\sqrt{100}$

= 10

17. (C) $x^2 + x - 2 = x^2 + 2x - x - 2$

$= x(x + 2) - 1\ (x + 2) = (x + 2)\ (x - 1)$

$x^2 - x - 6 = x^2 - 3x + 2x - 6$

$= x(x - 3) + 2\ (x - 3) = (x - 3)(x + 2)$

$x^2 - 4x + 3 = x^2 - 3x - x + 3$

$= x(x - 3) - 1(x - 3) = (x - 3)(x - 1)$

$\therefore \sqrt{(x^2+x-2)(x^2-x-6)(x^2-4x+3)}$

$=\sqrt{(x+2)(x-1)(x-3)(x+2)(x-3)(x-1)}$

$=\sqrt{(x+2)^2(x-1)^2(x-3)^2}$

$= (x - 1)(x + 2)(x - 3)$

18. (D) Here $p - 10^o + p - 5^o + p - 15^o + p - 30^o$

$= 180^o$

$\Rightarrow\ p=\frac{240^\circ}{4}=60^\circ$

19. (C) $4(x^2 + 10x + 25) - (4x^2 + 4x + 1)$

$= 3x - 15 + 180$

$4x^2 + 40x + 100 - 4x^2 - 4x - 1$

$= 3x + 165$

$36x + 99 = 3x + 165$

$33x = 66$

$x = 2$

20. (A) $(2P - 3q)\ (4p^2 + 6pq + 9q^2) + (2p + 3q)\ (4p^2 - 6pq + 9q^2)$

$= 8p^3 + 12p^2q + 18pq^2$

$-12p^2q - 18pq^2 - 27q^3 + 8p^3 - 12p^2q +$

$18pq^2 + 12p^2q - 18pq^2 + 27q^3 = 16\ p^3$

(OR) use $(a^3 - b^3) + (a^3 + b^3) = 2a^3$.

21. (C) $x + y = P\ (x + y + z)$

$y + z = P\ (x + y + z)$

$z + x = P(x + y + z)$

$\therefore\ x + y + y + z + z + x = 3P\ (x + y + z)$

$2\ (x + y + z) = 3\ P\ (x + y + z)$

$P=\frac{2}{3}$

22. (A) Length = 4x & breadth = 3x

$\therefore$ $12x^2 = 1728$

$x^2 = 144$

$x = 12$

$\therefore$ $l = 48$ mts & $b = 36$ mts

$P = 2(l + b) = 168$

Total cost of fencing = 168 m × ₹ 2.5

= ₹ 420

23. (C) Let speed of steamer be xKMPH

Let the distance be 'd' KM

Given $\frac{d}{x+2} = 4$

$d = 4(x + 2) = 4x + 8 \rightarrow (1)$

Given $\frac{d}{x-2} = 5$

$d = 5x - 10 \rightarrow (2)$

from (1) & (2) $4x + 8 = 5x - 10$

Speed of steamer (x) = 18 KMPH

24. (D) $1^3 + 12^3 = 10^3 + 9^3$ ie Both are equal to 1729.

$\therefore$ $\sqrt{x} = 3$

$x = 9$

25. (D) Given $xy = 45$ & $x - y = 4$

squaring on both sides

$x^2 + y^2 - 2yx = 16$

$x^2 + y^2 - 90 = 16$

$x^2 + y^2 = 106$

26. (C) Let the price of each article be 'x' number of articles sold be y then orignal sales amount = xy

New sales amount

$$= x\frac{(80)}{100} \times \frac{180}{100} y = \frac{36xy}{25}$$

Increased sales =

$$= \frac{36xy}{25} - xy = \frac{11xy}{25}$$

Increased sales percentage

$$= \frac{\left(\frac{11\cancel{xy}}{\cancel{25}}\right)}{\cancel{xy}} \times \cancel{100}^{4} = 44\%$$

27. (A) Given l : b : h = 3 : 2 : 1 = 3x : 2x : x

Given TSA = 88 cm^2

$2(6x^2 + 2x^2 + 3x^2) = 88\ cm^2$

$22x^2 = 88\ cm^2$

$x^2 = 4$

$x = 2$

$LSA = 2h\ (l + b) = 2(2)\ [6 + 4]$

$= 4 \times 10 = 40\ cm^2$

28. (C) Sum of length of edges = 12a

12 × 4 cm = 48 cm

29. (A) Area of rectangle

$= lb = (3p + 5q)(5p - 7q)$

$= 15p^2 - 21pq + 25pq - 35q^2$

$= 15p^2 + 4pq - 35q^2$

30. (D) $$\frac{\sqrt{72} \times \sqrt{363} \times \sqrt{175}}{\sqrt{32} \times \sqrt{147} \times \sqrt{252}}$$

$$= \frac{\sqrt{6 \times 6 \times 2} \times \sqrt{11 \times 11 \times 3} \times \sqrt{5 \times 5 \times 7}}{\sqrt{4 \times 4 \times 2} \times \sqrt{7 \times 7 \times 3} \times \sqrt{6 \times 6 \times 7}}$$

$$= \frac{6\sqrt{2} \times 11\sqrt{3} \times 5\sqrt{7}}{4\sqrt{2} \times 7\sqrt{3} \times 6\sqrt{7}}$$

$$= \frac{55}{28}$$

MATHEMATICS - 2 (MAQ)

31. (A, B) If a perfect square has 'n' digit its square root may have $\frac{n}{2}$ (or) $\frac{n+1}{2}$ digits.

32. (B, D) $(3^x)^2 - 10 \times 3^x + 9 = 0$

$(3^x)^2 - 9 \times 3^x - 3^x + 9 = 0$

$3^x(3^x - 9) - 1\ (3^x - 9) = 0$

$(3^x - 9)\ (3^x - 1) = 0$

$3^x - 9 = 0$	(or)	$3^x - 1 = 0$
$3^x = 9$		$3^x - 1$
$3^x - 3^2$		$3^x - 3^0$
$x = 2$		$x = 0$

33. (A, B, C, D)

If $\sqrt{xy}$ = 10, then (A) x = 20 & y = 5

(or)

x = 50 & y = 2,

x = 100 & y = 1

x = 25 & y = 4

34. (B, D) $x^2 + 2x - 15 = x^2 + 5x - 3x - 15$

$= x(x + 5) - 3(x + 5)$

$= (x + 5)(x - 3)$

$(x^2 - 2x - 35) = x^2 - 7x + 5x - 35$

$= x(x - 7) + 5(x - 7)$

$= (x - 7)(x + 5)$

$(x + 5) \mid (x^2 + 2x - 15), (x^2 - 2x - 35)$

$(x - 3), (x - 7)$

LCM of $(x^2 + 2x - 150$ and $(x^2 - 2x - 35) = (x + 5)(x - 3)(x - 7)$

$= (x - 3)(x + 5)(x - 7)$

$= (x - 3)(x^2 - 2x - 35)$

(or)

$(x - 7)(x + 5)(x - 3)$

$(x - 7)(x^2 + 2x - 15)$

35. (A, B, C, D)

$(2x^2 + x)^2 - 9(2x^2 + x) + 18 = (2x^2 + x)^2 - 6(2x^2 + x) - 3(2x^2 + x) + 18$

$= (2x^2 + x)[2x^2 + x - 6] - 3(2x^2 + x - 6)$

$= (2x^2 + x - 6)(2x^2 + x - 3)$

$= (2x^2 + 4x - 3x - 6)(2x^2 + 3x - 2x - 3)$

$= [2x(x + 2) - 3(x + 2)][x(2x + 3) - 1(2x + 3)]$

$= (x + 2)(2x - 3)(2x + 3)(x - 1)$

$(x + 2)$, $(2x - 3)$, $(2x + 3)$ & $(x - 1)$ are coprimes and factors of the given expression.

$\therefore$ Their product also factors of the given expression

$\therefore$ $(2x - 3)(2x + 3) = (4x^2 - 9)$ and $(x + 2)(x - 1) = x^2 - x + 2x - 2 = x^2 + x - 2$ (or)

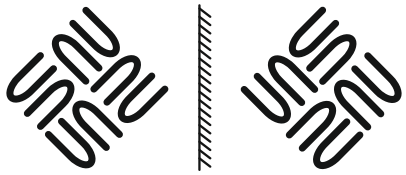
$(2x^2 + x - 3)$ is a factor of the given expression.

$(2x - 3)(x - 1) = 2x^2 - 2x - 3x + 3$

$= (2x^2 - 5x + 3)$ is also a factor of the given expression.

REASONING

36. (B)

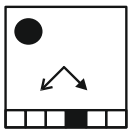


37. (D)

45, 42, 39, 36, 33, 30, (27), 24, 21, 18, 15, 12, 9, 6, 3

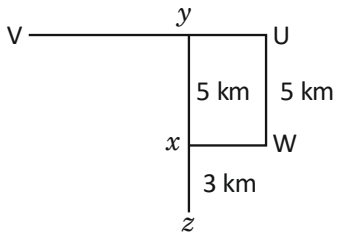
(27) is 9^{th}

38. (D)



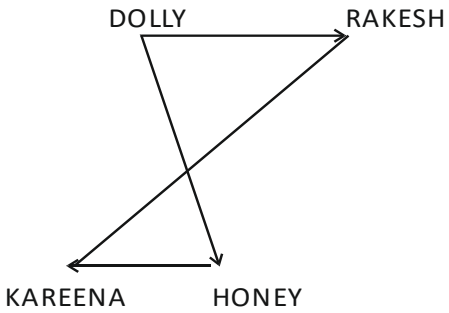
39. (B) The letter 'T' faces the base of the cube.

40. (A)



41. (B) In the first column $25 = (17 - 12)^2$ and second column $4 = (13 - 11)^2$, therefore $(19 - 16)^2$ is 9.

42. (C) Rakesh is to the north east of Kareena.

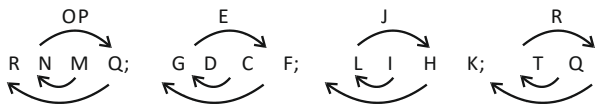


43. (A) CARROM → BZQQNL (−1) ; IPVTF → HOUSE (−1)

44. (C) $8^3 = 512 = 512 \div 2 = 256$

$10^3 = 1000 = 1000 \div 2 = 500$

45. (A) Except in option (A), in all other groups there is a gap of one letter as in the original alphabet between second and fourth letters.



CRITICAL THINKING

46. (B) When pulleys are used together in the way as shown in pulley B, they reduce the amount of force needed to lift weight or a load.

47. (A) Giridhar and his wife $\rightarrow$ 2 members

3 sons and their wives $\rightarrow$ 6 members

'2' sons have two childrens and '1' son has 3 childrens $\rightarrow$ 7 childrens.

1 unmarried daughter $\rightarrow$ 1

daughter and her son $\rightarrow$ 2

old aunt and son-in-law $\rightarrow$ 2

2 + 6 + 7 + 1 + 2 + 2 $\rightarrow$ 20 members

48. (C) Plants are watered on wednesday.

49. (C) All Violinist are instrumentatlists. All instrumentalists are musicians. Then the relationship is correctly represented in option (C).

50. (C) Squares move from the top left to the middle, then to the bottom right corner with each turn. Unshaded squares appear in every third turn. Triangle appears with every alternate turn.

Therefore, the black square and unshaded square should both be in the middle, with no triangle.

The End